Field theoretical methods in fluid and plasma theory

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Experimental, numerical and theoretical studies have revealed that the two-dimensional fluids and plasmas exhibit an intrinsic evolution to organization. This is most obvious at relaxation from turbulent states when the system evolves toward a reduced subset of flow patterns, characterized by a regular form of the streamfunction (*coherent structures*). Although in many situations the fluids and plasmas are subject to strong driving/dissipative processes (which may overcome and hide the self-organization tendency) the fact that the ideal system selects a restricted set of states rises the problem of *preferred states* in fluid and plasmas. This problem is common to several systems: ideal incompressible (Euler) fluid, plasma in strong magnetic field, planetary atmosphere, non-neutral plasmas, etc. *It is admitted that we do not dispose, at this moment, of a satisfactory understanding of this fundamental process.*

The conservation equations (treated as dynamical equations) are not appropriate, we need a variational approach, the preferred states being determined as configurations of flow that extremize an action functional. For the $2D$ case there exist models that are equivalent to the physical systems: the $2D$ ideal (Euler) fluid and the $2D$ magnetized plasma (and planetary atmosphere) are equivalent with discrete systems consisting of sets of point-like vortices interacting in plane by a potential (Coulombian and respectively short range). Looking for a formulation of the continuum limit of these discrete models we have been naturally led to classical field theories. The matter field (the density of the point-like objects : vortices or currents) is a complex field with a scalar self-interaction, the potential of interaction between these objects is represented by a gauge field with Chern-Simons action, the coupling is minimal.

We will present the construction of Lagrangian density for the Euler fluid. The theory is in $sl(2, \mathbf{C})$ which reflects the spinorial nature of the point-like vortices. The matter field nonlinear interaction is of order four. We apply the Bogomolnyi procedure writting the action as a sum of square terms plus (possibly) a term with topological content, and identify a new set of (self-duality) equations describing the stationary states. It results at self-duality the equation *sinh*-Poisson, known to govern the asymptotic ordered states. It must be noted that the self-duality is the property which is at the origin of all known exactly integrable nonlinear equations and of their coherent nonlinear structures, like solitons, instantons, vortices of the Abelian-Higgs superconductors, Painleve transcendents, etc. We will derive the equations of motion, the expression of the (field theoretical) current and examine the states in close proximity of the self-duality. The form of the energy functional close to self-duality appears to be only weakly selective for minimal states, which suggests that reaching the relaxed states the system slows down and also can have various metastable states. A series of numerical solutions will be presented in support to this qualitative conclusion.

It will be shown that the stationary states obtained at relaxation from turbulence (*i.e.* solutions of the *sinh*-Poisson equation) can also be identified from a zero-curvature condition applied on a new gauge potential defined from a combination of the basic gauge and matter variables. The physical meaning of the field theoretical variables is discussed.

In studying the model for the Euler equation we will frequently make reference, for simplicity, to the closely related Abelian field theoretical model for the continuum limit of the point-like currents interacting in plane via Coulombian potential. This leads at stationarity to the Liouville equation, describing the current profile in tokamak. A short mention will be made for similar models for plasma in strong magnetic field and for the planetary atmosphere.