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## FOREWORD

A large fraction of the worldwide efforts to demonstrate the feasibility of obtaining fusion energy by magnetically confined plasmas is centered around large tokamak devices and, in a smaller scale, helical systems. This narrowing of the research scope has had the adverse effects by putting down all the efforts of university laboratories and laboratories in the third world countries to pursue the fusion research. There is a growing movement to (1) encouraging research on other known devices which are not yet entirely explored, (2) encouraging initiatives to explore new ideas, (3) encouraging efforts to study the details of tokamak physics using small devices, (4) encouraging fusion research in countries with smaller financial resources, and finally (5) encouraging the upbringing of young researchers in this field.

The main objective of the IAEA Technical Committee Meeting on Research Tokamaks is to provide a forum to review the research efforts going on in all the small devices scattered around the world. This forum is also the proper place to start discussing a worldwide collaborative research effort using small devices.

The realization of this Meeting had a relative success, as shown in fine details by the review done expertly by Dr. Tom Todd, from Culham Laboratory.

Our thanks for all the participants who made this meeting an important event in South America, especially to Dr. Todd for his review.

These proceedings contains manuscripts which were retyped excellently by Rosa Cristiane F. G. Jordão. Our thanks for her expert work.

Campinas, August 1, 1996.

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# STATIONARY TURBULENT STATES OF COMPETING INSTABILITIES

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## 1 Abstract

The stationary plasma state at the tokamak edge is represented as the alternance of two types of instabilities in the strong turbulent regime. An analytical description is proposed for the case of the dissipative drift wave turbulence replaced intermittently by the resistivity gradient driven turbulence.

## 2 Introduction

The transport at the tokamak plasma edge is determined by the strong turbulent evolution of various instabilities[1]. The frequency and wavenumber spectra show that the fluctuations have, on the average, the diamagnetic frequency ( $\omega \sim \omega_*$ ) and long wavelength ( $k_\theta < 5\text{cm}^{-1}$ ) and also show that they have strong incoherent character, with large broadening in both  $\omega$  and  $k$ -space ( $\delta\omega \sim \omega, \delta K_\theta \sim K$ ). Two types of electrostatic instabilities have frequently been invoked to explain these features: dissipative drift wave instability[2] and resistivity gradient driven instability (modified by including the radiative cooling and the gradient of the effective charge)[3,4]. Comprehensive theoretical descriptions are available for these instabilities. When the theoretical results are compared to the measurements[5] one is led to conclude that none of these particular models is able to explain the full set of experimental characteristics. Only a certain "combination" of theoretical predictions from both instabilities would provide a satisfactory picture.

The idea which we propose in this work is to consider the following picture: the drift wave turbulence and the  $\nabla\eta$ -turbulence alternatively dominate the dynamics of the plasma in the finite region of interest and they replace one another in a series of switches which apparently is random. The measured plasma turbulence should consist in a superposition of the individual characteristics, projected out through this series of jumps.

This is often the case for dynamical systems: if the phase space is divided in two attractors, the system's evolution consists of a sequence of "laminar" states representing motion on one or another of attractors, interrupted by jumps between them.

### 3 Individual characteristics of the two types of turbulence

The dissipative drift instability is driven by the relaxation of the equilibrium density gradient and is destabilized by electron collisions. The basic nonlinear treatment [2] uses the continuity and the parallel momentum balance for the inertialess electrons. The linear part in the resulting equation for the nonadiabatic component of the density fluctuation consists in the curvature drift and the parallel viscous diffusion and the nonlinear part is due to the  $E \times B$  convection of the fluctuation by its own field. The gradient-of-resistivity-driven instability is described by the Ohm's law, the vorticity equation and the equation for the evolution of the resistivity. The strong stabilizing mechanism is the parallel thermal diffusion. For both instabilities analytical treatment of the strong turbulence regime exists, based on the direct-interaction-approximation-renormalization of the triplet nonlinearity appearing in the equation for the two-point correlation [2,3]. This results in the effective replacement of the nonlinearity by a diffusion operator with the coefficient inhomogeneous in the relative coordinate. The correlations have a finite life-time  $\tau_d(x_\perp, y_\perp, z_\perp)$  because the spectrum of fluctuations contains few waves with very short wavelength which could discriminate (and separate) two initially very close fluid elements. This approach permitted to identify the possibility of the state of saturated  $\nabla\eta$ -driven turbulence (as opposed to the linear prediction) in the form of the "decoupling condition" - a relation between the potential  $\phi$  and resistivity  $\eta$  fluctuations. It implies that  $\phi$  and  $\eta$  decouple from the current density fluctuations and that the strong stabilizing linear mechanism of  $\chi_{II}$  is much less effective. A spatial scale of the  $\nabla\eta$ -turbulence results from this condition [3]:

$$\Delta_k = \left( \frac{E_0 L_s}{B_0 L_n} \right)^{1/3} \left( \frac{\chi_{II} K^2}{L_s^2} \right)^{-1/3} \quad (1)$$

If the spatial extension of the potential correlation is  $\Delta_k$  the  $\nabla\eta$ -turbulence can saturate leading to a diffusion coefficient with a weak dependence on  $\chi_{II}$ :  $D \sim \chi_{II}^{-1/3}$ . The existence of a condition for the onset of  $\nabla\eta$ -turbulence suggests a slightly different perspective on the alternance of the two types of instabilities: we shall assume that the drift wave turbulence is a background, with intermittent switches to  $\nabla\eta$ -turbulence.

### 4 Statistical approach to intermittency

Following a general treatment [6] we consider a time series of symbols  $s_i \in \{0, 1\}$  and the probability of a sequence  $p(s_1, \dots, s_n)$ . In particular  $\omega_j(s, s') = s, s, \dots, s, s'$  consists of  $j-1$  states of types followed by a state  $s'$  and we introduce the conditional probability  $p(s/\omega_j(s, s')) = t_{ss'} \exp(-W_s(j))$  for  $s = 0, s' = 1$ , and  $p(s/s') = t_{ss'}$  for  $s = 1$ . The main idea is to construct and equivalent model of a gas of interacting particles on a 1-D lattice. The interaction energy  $U$  is defined by:  $\exp(-U(s, s'))$

$j) = t_{ss'} \exp(-W_s(j))$ , for  $s = 0, s' = 1$ ;  $= 0$ , for  $s = s' = 0$ ; and  $t_{ss'}$ , for  $s = 1$ . A partition function  $Z_{n,m}(q) \equiv \sum_{s=0}^1 \Omega_m(P, q)$  and a generating functional

$$\Xi(P, q) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e^{-Pn} Z_{n,m}(q) \equiv \sum_{n=0}^{\infty} \Omega_m(P, q) \quad (2)$$

are introduced in order to calculate the average of a quantity  $Q$ :

$$\langle Q \rangle = (\Xi(P, q))^{-1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e^{-Pn} Q Z_{n,m}(q) \quad (3)$$

The explicit expression for  $\Omega_m(s)$  can be obtained considering  $m$  "particles"  $s_i$  at sites  $i$  on an available "space" of  $n$  sites, and with relative distances  $j_i$ . Then  $\Omega_m(P, q) = \text{tr}(T^m)$ , with the matrix  $T$  defined by:

$$\begin{aligned} (t_{ss'})^q &= \sum_{j=1}^{\infty} e^{Pj - qW_s(j)}, \text{ for } s = 0, s' = 1 \\ < s | T | s' > = 0, \text{ for } s = s' = 0 \\ (T_{ss'})^q &= e^{-P}, \text{ for } s = 1 \end{aligned} \quad (4)$$

It results:  $\Xi(P, q) = 1/(1 - \lambda_1) + 1/(1 - \lambda_2)$ , ( $\lambda_i$  are the eigenvalues of  $T$ ). We choose (like in [6])  $p(0 | \omega_j(0, 1)) \sim j^{-r}$ , ( $r > 1$ ) which gives:  $p(0 | \omega_j(0, 1)) = c \exp(-W(j))$ ,  $p(0, 1) = a$ ,  $p(1, 1) = b$ , with  $a + b = 1$  and  $W(j) = r \ln j$ . This allows us to obtain an explicit expression for the function  $\Psi(q)$  as  $\lim_{c \rightarrow 0} \langle n \rangle < m_1 / \langle n \rangle$ , where  $m_1$  is the number of "particles" of type 1":

$$\Psi(q) \approx [1 + a\zeta(r-1)]^{-1} + (1 - q) \quad (5)$$

for  $r > 3$  ( $\omega$  is a positive constant),  $\zeta$  is the Riemann function. The function  $\Psi(q)$  is an order parameter, expressing the fraction occupied by phase "1" in the evolution.  $q$  has the formal meaning of a temperature, its variation allowing us to obtain states which are dominated, for example, by the phase "1".

### 5 Physical mechanism at the onset of the intermittency

As showed in [3]  $\nabla\eta$ -turbulence exists only when the "decoupling condition" is fulfilled:

$$\chi_{II} K_{\theta}^2 B_0^2 / L_s^2 > D_K / \Delta_K^2 \approx E_0 L_s / (L_n B_0 \Delta_K) \quad (6)$$

(The notations are those of [3]). The radial space scale  $\Delta_K$  is required. Also, at saturation, the time scale of the power transfer is defined by:

$$\tau_{\nabla\eta} = L_n B_0 \Delta_K / (L_s E_0) \quad (7)$$



The drift wave turbulence makes possible these values (in the particular range of Reynolds number of order unity, - compatible with the saturated state of  $\nabla\eta$ -turbulence). The slow time scale of the drift wave turbulence evolution should permit the existence of a real solution of the equation:

$$\tau_{cl}^{drift}(\Delta_{kl}R_e) = \tau_{\nabla\eta} \quad (8)$$

or

$$\frac{L_z B_0 \Delta_K}{L_S E_0} = \frac{1}{K_\theta^2 D} \ln(K_\theta^{-2}(\tilde{S}^2 X_-^2 + \frac{\eta_-^2}{K_\theta^2 \Delta\eta_c^2} + \frac{1}{K_\theta^2 R_e})^{-1} \quad (9)$$

for  $X_- = \Delta_K$ . Here  $R_e = D_K^2(Rq)^2(\Delta\eta_c)^2 D_{II}^{-1}$ , [2]. For usual magnitudes of the parameters above, we obtain  $\Delta_K \simeq 1$  cm. The  $\nabla\eta$ -turbulence theory provides a similar value  $\Delta_K \sim 1$  also confirmed by the numerical simulations.

In conclusion, the drift wave turbulence can provide, in particular regimes, the parameters required for the "decoupling condition" of  $\nabla\eta$ -turbulence. The drift wave turbulence is always close to such a regime and it can be driven in it (thus making possible a real solution to the equation above) by the slow variation of the local equilibrium parameters.

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